A Catalog of Recommended Multiple Comparison Procedures

Table 10.6 Recommended procedures for controlling *FWE*

Family type	Simultaneous methods[a]		Sequential methods	
	Equal variances	Unequal variances	Equal variances	Unequal variances
Planned	Dunn–Bonferroni	Dunn–Bonferroni using Welch's t'	Hochberg	Hochberg using Welch's t'
All pairwise	Tukey *HSD* (equal n) or Tukey–Kramer (unequal n)[b]	Games–Howell or Dunnett T3	Fisher–Hayter	
Exptl vs control	Dunnett (equal n) or Dunn–Bonferroni (unequal n)	Dunn–Bonferroni using Welch's t'		
Post hoc	Scheffé	Scheffé using Welch's t'		

[a] Only the simultaneous methods allow the construction of simultaneous confidence intervals.

[b] Assuming K pairwise tests, the Bonferroni method will be more powerful than the Tukey under some conditions; Equation 10.19 provides the basis for the choice.

1. **Standard *t*-test**. This is the general procedure for testing the hypothesis that

$$\Psi = \sum_{j=1}^{a} c_j \mu_j = 0$$

with a test statistic of the form

$$t_{n_\bullet - a} = \frac{\hat{\Psi}}{\sqrt{\hat{\sigma}^2_{\hat{\Psi}}}} = \frac{\sum_{j=1}^{a} c_j \overline{X}_{\bullet j}}{\sqrt{\left(\sum_{j=1}^{a} \frac{c_j^2}{n_j}\right) MS_{error}}}$$

The critical value is taken from Student's *t*-distribution with degrees of freedom equal to $n_\bullet - a$.
For two contrasts to be *orthogonal*, they must satisfy the restriction that the sum of cross-products of their linear weights must be zero, i.e., $\sum_{j=1}^{a} c_{1j} c_{2j} = 0$

2. Welch t' **statistic**. This revised test statistic does not assume equal variances.

$$t'_{v'} = \frac{\hat{\Psi}}{\sqrt{\hat{\sigma}^2_{\hat{\Psi}}}} = \frac{\sum_{j=1}^{a} c_j \overline{X}_{\bullet j}}{\sqrt{\sum_{j=1}^{a} \frac{c_j^2 s_j^2}{n_j}}}$$

The degrees of freedom are modified to take into account unequal variances. **Welch's** formula for the adjusted degrees of freedom is

$$\nu' = \frac{\left(\sum_{j=1}^{a} \frac{c_j^2 s_j^2}{n_j} \right)^2}{\sum_{j=1}^{a} \frac{c_j^4 s_j^4}{n_j^2 \left(n_j - 1 \right)}}$$

3. **Dunnett's test.** This is simply the standard *t*-statistic (1), compared to special critical values given in, for example, Table M of Glass and Hopkins or Table C-8 of RDASA3. The test requires equal *n*.

4. **Tukey's HSD test.** This test compares pairwise mean differences against a single value, called the HSD. The HSD is calculated as

$$HSD = q^*_{a,n_{\bullet}-a} \sqrt{\frac{MS_{error}}{n}}$$

5. The **Tukey-Kramer modification** for unequal *n* uses

$$HSD = q^*_{a,n_{\bullet}-a} \sqrt{\left(\frac{1}{2n_1} + \frac{1}{2n_2} \right) MS_{error}}$$

6. **Games-Howell test.** This is, in essence, the Tukey-Kramer test with a Welch-type modification. One uses a (potentially) different HSD for each pairwise contrast. The formula is.

$$HSD = q^*_{a,\nu'} \sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}$$

where ν' is the Welch-adjusted degrees of freedom given in (2) above.

7. **Scheffé test**. This procedure allows all contrasts to be performed with a familywise error rate of α. It is extremely conservative. One performs the standard *t*-test, but compares it to the critical value

$$S = \sqrt{(a-1) F^*_{\alpha, J-1, n_{\bullet}-a}}$$

8. Brown-Forsythe test. Like the Scheffé test, but uses t' in lieu of *t*, and the Welch-corrected ν' in lieu of $n_{\bullet} - J$.

9. **Fisher-Hayter test.** This two-stage procedure begins with an omnibus *F* test. If this rejects, then each pairwise contrast is tested with a statistic

$$q_{\alpha,a-1,n_{\bullet}-a} = \frac{\overline{X}_{\bullet i} - \overline{X}_{\bullet j}}{\sqrt{\left(\frac{1}{n_i} + \frac{1}{n_j} \right) \frac{MS_e}{2}}}$$